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Ginzburg–Landau functional for type-II superconductors with Pauli paramagnetic effect

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Abstract
Effects of Pauli paramagnetism on type-II superconductivity are examined through a derivation of a Ginzburg–Landau (GL) functional by including the field-induced pair-breaking effects through the spin and orbital on an equal footing. On the basis of the resultant GL functional we discuss the corresponding high-field vortex phase diagram at the level of mean field approximation. Special attention is paid to the mutuality of characteristic temperatures, i.e., the temperatures: (1) at which the vortex state is described in terms of higher Landau levels; (2) at which a vortex state modulated along the field direction (a Fulde–Ferrell–Larkin–Ovchinnikov-like state) becomes stable; and (3) at which a superconducting transition becomes *nearly* discontinuous.

1. Introduction

Since sample inhomogeneity can easily weaken the characteristic effects of paramagnetic depairing on type-II superconductivity, a very clean sample which simultaneously possesses a high orbital limiting field is needed to study such effects. The recently discovered heavy-fermion superconductor $CeCoIn_5$ [1], because of its high purity, high transition temperature, and strong renormalization of the Fermi velocity, provides a realistic situation where we must consider a vortex state with a strong paramagnetic depairing. In fact the shape of the H_{c2}-line and *nearly* discontinuous behaviours at H_{c2} seen in transport and thermodynamic quantities [2] imply the existence of a strong paramagnetic effect. A reliable theoretical tool for explaining the above phenomena is Ginzburg–Landau (GL) theory. Conventionally a microscopic derivation of a GL functional has always been performed under the assumption that we can treat the effect of orbital depairing perturbatively [4, 5]. Unfortunately this assumption breaks down in the region of interest because a behaviour peculiar to the paramagnetic depairing emerges under the condition $I \gtrsim \tau_B^{-1} > T$ where $I = \mu_0 B$ is the Zeeman energy, $\tau_B = r_B/v_F$, and $r_B = \sqrt{\phi_0/2\pi B}$ is the magnetic length. We also have to take impurity scattering into account

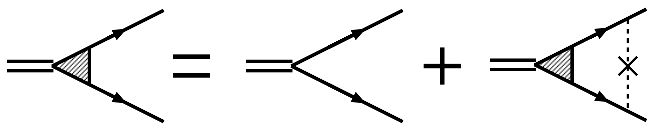

Figure 1. Diagrams representing the impurity vertex correction.

since, as one can see below, even a weak impurity is relevant to the discussion of a phase diagram for *real* materials.

In this paper we report the result of a microscopic calculation of a GL functional which, together with weak impurity scattering, includes the effects of orbital and of paramagnetic depairing on an equal footing. Our aim here is to evaluate each term of the following GL functional:

$$F = N(0)\int_{r_\perp}\left\{\sum_{q_z}\sum_{N} a_N(q_z^2)|\tilde{\Delta}_{q_z}^{(N)}|^2 + \sum_{l}\left(\frac{b}{2}|\Delta_l^{(0)}|^4 + \frac{c}{3}|\Delta_l^{(0)}|^6\right)\right\}, \quad (1)$$

where $\Delta_l^{(N)} = N_{\mathrm{layer}}^{-1/2}\sum_{q_z}\tilde{\Delta}_{q_z}^{(N)}\mathrm{e}^{\mathrm{i}q_z s l}$ denotes the pair field (superconducting order parameter) in the Nth Landau level, $N(0)$ is the density of states at the Fermi surface. Some comments are needed on the form of the GL functional. Firstly, having CeCoIn_5 in mind we assume a quasi-two-dimensional system. Secondly, we restrict the fourth- and sixth-order terms to the lowest Landau level (LLL), because in a realistic situation a wider region described by the LLL pair field does exist. Finally, we approximate the fourth and sixth couplings in equation (1) as local ones, although the correct forms of these couplings are *non-local*. Details of the validity of this approximation and a more complete treatment will be given in a future publication [8].

2. Quadratic term

Assuming a quasi-two-dimensional BCS Hamiltonian and treating the effect of the magnetic field quasi-classically, the coefficient $a_N(q_z^2)$ in equation (1) has the following form:

$$a_N(q_z^2) = \ln(T/T_{c0}) - 2\pi T\sum_{\varepsilon>0}(D_N(\varepsilon) - 1/\varepsilon), \quad (2)$$

$$D_N(\varepsilon) = 2\int_0^\infty \mathrm{d}\rho\,\exp\left(-2\varepsilon\rho - \left(\frac{\rho}{2\tau_B}\right)^2\right)\mathcal{L}_N\left(2\left(\frac{\rho}{2\tau_B}\right)^2\right)\cos(2I\rho)\mathcal{J}_0\left(2J\sin\left(\frac{q_z s}{2}\right)\rho\right). \quad (3)$$

Here J and s are the interlayer hopping and spacing, $\mathcal{L}_N$ and $\mathcal{J}_N$ are the Nth-order Laguerre and Bessel functions respectively. Throughout this paper we assume that a magnetic field is always applied perpendicular to the conducting plane. In order to take account of impurity scattering, we have to include the finite lifetime τ of quasi-particles as well as the impurity vertex correction shown in figure 1. This procedure corresponds to replacing $D_N(\varepsilon)$ in $a_N(q_z^2)$ by $D_N(\varepsilon+\Gamma)/(1-\Gamma D_N(\varepsilon+\Gamma))$ where $\Gamma = 1/2\tau$.

Figure 2 shows an H_{c2}-line determined by the condition $a_N(0) = 0$ for a moderately strong paramagnetic depairing $\mu_0 H_{c2}^{\mathrm{orbit}}(0)/2\pi T_{c0} = 0.85$ where $H_{c2}^{\mathrm{orbit}}(0) = 0.089\phi_0/\xi_0^2$ is a two-dimensional orbital limiting field. In a low-temperature region, as was first pointed out by Buzdin [3], $H_{c2}(T)$ corresponding to the next Landau level (NLL) exceeds the one corresponding to the LLL at some temperature T_{next}. From the figure one can see that this temperature is easily suppressed by a weak impurity scattering (figure 2(b)). Incidentally, under the same conditions, there exists another temperature below which a modulated vortex

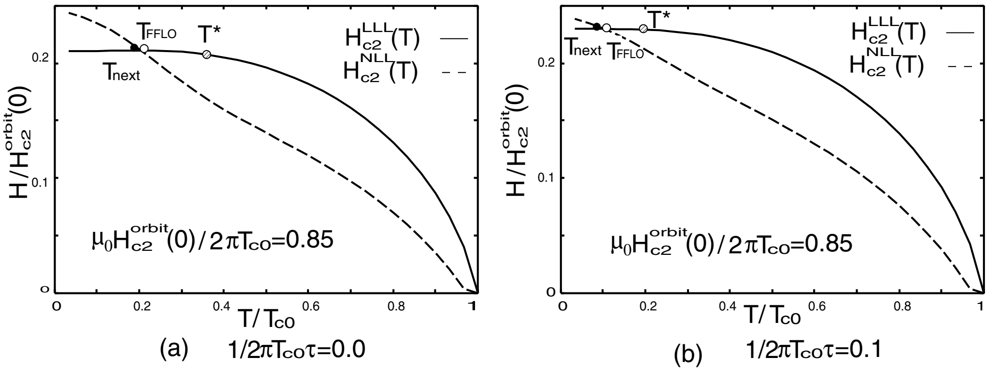

Figure 2. The H_{c2}-lines corresponding to LLL and NLL. For the definition of the temperatures T_{next}, T_{FFLO}, and T^*, see the text.

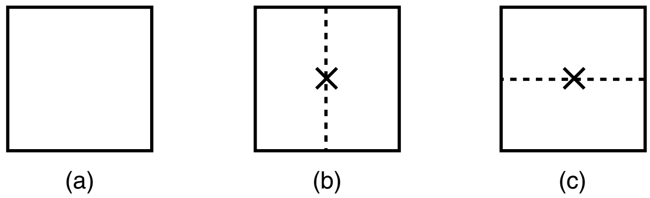

Figure 3. Diagrams contributing to the quartic term in equation (1).

state (a Fulde–Ferrell–Larkin–Ovchinnikov-like (FFLO-like) state) becomes stable. We can examine this temperature by calculating the lowest gradient term along a field, and this temperature T_{FFLO} is also plotted in figure 2. In the parameter regions studied, we find that both temperatures are always located close to each other.

3. Higher-order term

Next we evaluate the quartic term in equation (1). As Mineev [4] demonstrated, this term can be positive or negative, and in the case of a negative quartic term, resultant phase transition becomes *nearly* discontinuous. However, it is quite important to note that a negative quartic term does not in itself lead to a true first-order phase transition at H_{c2} immediately. This point was explained in [6, 8]. Three diagrams shown in figure 3 contribute to the quartic term. Because the high-energy part in the Matsubara summation gives a lower contribution, we can neglect the impurity vertex correction represented in figure 1 for the clean limit. The expression for the coefficient $b = b_a + b_{bc}$ is given by

$$b_a = 2\pi T \int \prod_{i=1}^{3} \mathrm{d}\rho_i \, f\left(\sum_{i=1}^{3} \rho_i\right) \langle I_4(\rho_1\rho_2\rho_3, 0|\zeta\zeta)\rangle_\zeta + \text{c.c.}, \tag{4}$$

$$b_{bc} = -\frac{2\pi T}{\tau} \int \prod_{i=1}^{4} \mathrm{d}\rho_i \, f\left(\sum_{i=1}^{4} \rho_i\right) \langle I_4(\rho_1\rho_2\rho_3\rho_4|\zeta\xi)\rangle_{\xi,\zeta} + \text{c.c.}, \tag{5}$$

where the complex numbers ζ, ξ with unit length denote the position on the Fermi surface, and the functions f and I_4 are defined by

$$f(\rho) = \exp(-\rho/\tau)\cos(2I\rho)/\sinh(2\pi T\rho), \tag{6}$$

$$I_4(\rho_1\rho_2\rho_3\rho_4|\zeta\xi) = \exp(-\{|\vec{\rho}|^2 + \tfrac{1}{2}[(\rho_1\zeta - \rho_3\xi)^2 + (\rho_2\zeta - \rho_4\xi)^{*2}] + (\rho_1\zeta + \rho_3\xi)(\rho_2\zeta + \rho_4\xi)^*\}/4\tau_B^2), \qquad \text{where } |\vec{\rho}|^2 = \sum_{i=1}^{4}\rho_i^2. \tag{7}$$

Through a numerical calculation we work out a temperature T^* below which the coefficient b becomes negative, and the temperature is also plotted in figure 2. The result shows that this temperature is always higher than the previous two temperatures. Finally, the stability of the GL functional is inspected, i.e., the coefficient c in equation (1) always has a positive sign in the region where b is negative.

4. Summary

In summary we have studied a GL functional microscopically and found that

(i) T_{next} and T_{FFLO} are located close to each other;
(ii) T^* is higher than T_{next} or T_{FFLO}; and
(iii) the sixth-order term in equation (1) is positive in the region of negative quartic terms.

These facts mean that a *nearly* discontinuous behaviour at H_{c2} seen for $CeCoIn_5$ can be discussed on the basis of a GL functional, used in [6, 8], with a negative quartic term and a positive sixth-order term within the LLL subspace. The theoretical argument in [6, 8] is based on the fact that the correct phase diagram for a vortex state must be discussed including fluctuation effects [7] even in a less fluctuating system and even in the case with a strong paramagnetic effect, because in a vortex state a characteristic dimensional reduction due to Landau quantization is always effective. Therefore the theoretically correct interpretation of the above phenomena is that the *nearly* discontinuous behaviour is just a (quite steep) crossover and this crossover has sometimes been misunderstood as a true first-order phase transition.

At this stage it is not clear whether T_{next} and T_{FFLO} for $CeCoIn_5$ are of experimentally accessible order. The theoretical description of a vortex state in such a region is left to future studies.

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